

Vježbe I

Brojni sistemi; konverzija jednog u drugi brojni sistem

Dekadni brojni sistem: osnova 10, cifre {0, 1, 2, ..., 9}

$$\text{npr. } 136_{(10)} = 1 \times 10^2 + 3 \times 10^1 + 6 \times 10^0$$

Binarni brojni sistem: osnova 2, cifre {0, 1}

$$\text{npr. } 1011_{(2)} = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

Oktalni brojni sistem: osnova 8, cifre {0, 1, 2, 3, 4, 5, 6, 7}

$$\text{npr. } 256_{(8)} = 2 \times 8^2 + 5 \times 8^1 + 6 \times 8^0$$

Heksadecimalni brojni sistem: osnova 16, cifre {0, 1, 2, ..., 8, 9, A, B, C, D, E, F}

$$\text{npr. } AC23_{(16)} = 10 \times 16^3 + 12 \times 16^2 + 2 \times 16^1 + 3 \times 16^0$$

1. Dat je broj 1044.75 u dekadnom brojnom sistemu. Pretvoriti ga u binarni brojni sistem.

		ostatak			cijeli dio		
1044	: 2 =	522	0	0.75	× 2 = 1.5	1	↓
522	: 2 =	261	0	0.5	× 2 = 1.0	1	↓
261	: 2 =	130	1				
130	: 2 =	65	0				
65	: 2 =	32	1				
32	: 2 =	16	0				
16	: 2 =	8	0				
8	: 2 =	4	0				
4	: 2 =	2	0				
2	: 2 =	1	0				
1	: 2 =	0	1				

$$1044_{(10)} = 10000010100_{(2)}, 0.75_{(10)} = 0.11_{(2)} \Rightarrow 1044.75_{(10)} = 10000010100.11_{(2)}$$

*** konverzija sa zadatom tačnošću

npr. 5 decimala

			cijeli dio
0.075	× 2 =	0.150	0
0.150	× 2 =	0.3	0
0.3	× 2 =	0.6	0
0.6	× 2 =	1.2	1
0.2	× 2 =	0.4	0...

$0.075_{(10)} = 0.00010..._{(2)}$

2. Dat je broj 10001011.1101 u binarnom brojnom sistemu. Pretvoriti ga u dekadni brojni sistem.

$$\begin{aligned} 10001011.1101_{(2)} &= 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} = \\ &= 2^7 + 2^3 + 2^1 + 2^0 + 2^{-1} + 2^{-2} + 2^{-4} = 128 + 8 + 2 + 1 + 0.5 + 0.25 + 0.0625 = 139.8125_{(10)} \end{aligned}$$

3. Dat je broj 1045 u dekadnom brojnom sistemu. Pretvoriti ga u oktalni brojni sistem:

- a) direktnom konverzijom,
 b) konverzijom u binarni, a potom u oktalni brojni sistem.

a)

$$\begin{array}{r}
 1045 : 8 = 130 \quad 5 \quad \uparrow \\
 130 : 8 = 16 \quad 2 \\
 16 : 8 = 2 \quad 0 \\
 2 : 8 = 0 \quad 2 \\
 1045_{(10)} = 2025_{(8)}
 \end{array}$$

b)

$$\begin{array}{r}
 1045 : 2 = 522 \quad 1 \quad \uparrow \\
 522 : 2 = 261 \quad 0 \\
 261 : 2 = 130 \quad 1 \\
 130 : 2 = 65 \quad 0 \\
 65 : 2 = 32 \quad 1 \\
 32 : 2 = 16 \quad 0 \\
 16 : 2 = 8 \quad 0 \\
 8 : 2 = 4 \quad 0 \\
 4 : 2 = 2 \quad 0 \\
 2 : 2 = 1 \quad 0 \\
 1 : 2 = 0 \quad 1
 \end{array}$$

$$1045_{(10)} = \overset{210210210210}{\underline{010\ 000\ 010\ 101}}_{(2)} = \overset{2025}{\underline{2\ 0\ 2\ 5}}_{(8)}$$

4. Dat je broj 345 u dekadnom brojnom sistemu. Pretvoriti ga u heksadecimalni brojni sistem:

- a) direktnom konverzijom,
 b) konverzijom u binarni, a potom u heksadecimalni brojni sistem.

a)

$$\begin{array}{r}
 345 : 16 = 21 \quad 9 \quad \uparrow \\
 21 : 16 = 1 \quad 5 \\
 1 : 16 = 0 \quad 1 \\
 345_{(10)} = 159_{(16)}
 \end{array}$$

$$\begin{array}{r}
 345 : 2 = 172 \quad 1 \\
 172 : 2 = 86 \quad 0 \\
 86 : 2 = 43 \quad 0 \\
 43 : 2 = 21 \quad 1 \\
 21 : 2 = 10 \quad 1 \\
 10 : 2 = 5 \quad 0 \\
 5 : 2 = 2 \quad 1 \\
 2 : 2 = 1 \quad 0 \\
 1 : 2 = 0 \quad 1
 \end{array}$$

$$345_{(10)} = \overset{321032103210}{\underline{0001\ 0101\ 1001}}_{(2)} = \overset{159}{\underline{1\ 5\ 9}}_{(16)}$$

5. Dat je broj 251 u oktalnom brojnom sistemu. Pretvoriti ga u binarni, heksadecimalni i dekadni brojni sistem.

$$251_{(10)} = \underline{010\ 101\ 001} = \underline{1010\ 1001}_{(2)} = A9_{(16)} = 10 \times 16^1 + 9 \times 16^0 = 169_{(10)}$$

6. Dat je broj AF12.3F u heksadecimalnom brojnom sistemu. Pretvoriti ga u binarni, oktalni i dekadni.

$$\begin{aligned}
 AF12.3F_{(16)} &= \underline{1010\ 1111\ 0001\ 0010} . \underline{0011\ 1111}_{(2)} = \\
 &= \underline{001\ 010\ 111\ 100\ 010\ 010} . \underline{001\ 111\ 110}_{(2)} = \\
 &= \underline{1\ 2\ 7\ 4\ 2\ 2} . \underline{1\ 7\ 6}_{(8)} =
 \end{aligned}$$

$$= 2^{15} + 2^{13} + 2^{11} + 2^{10} + 2^9 + 2^8 + 2^2 + 2 + 2^{-3} + 2^{-4} + 2^{-5} + 2^{-6} + 2^{-7} + 2^{-8} =$$

$$= 44818.24609_{(10)}$$

ili

$$AF12.3F_{(16)} = 10 \times 16^3 + 15 \times 16^2 + 1 \times 16^1 + 2 \times 16^0 + 3 \times 16^{-1} + 15 \times 16^{-2} = 44818.24609_{(10)}$$

ili

$$127422.176_{(8)} = 1 \times 8^5 + 2 \times 8^4 + 7 \times 8^3 + 4 \times 8^2 + 2 \times 8^1 + 2 \times 8^0 + 1 \times 8^{-1} + 7 \times 8^{-2} + 6 \times 8^{-3} = 44818.24609_{(10)}$$

Binarna aritmetika: sabiranje, oduzimanje, množenje, dijeljenje

Pravila sabiranja u binarnom brojnem sistemu:

0 + 0 = 0	Prenos:
0 + 1 = 1	0
1 + 0 = 1	0
1 + 1 = 0	1
1 + 1 + 1 = 1	1

7. Sabrati binarne brojeve i izvršiti provjeru u dekadnom brojnem sistemu.

a)

$\begin{array}{r} 10110 \\ + 11001 \\ \hline 101111 \end{array}$	$\begin{array}{r} 2^4 + 2^2 + 2^1 = 22_{(10)} \\ + 2^4 + 2^3 + 2^0 = 25_{(10)} \\ \hline 2^5 + 2^3 + 2^2 + 2^1 + 2^0 = 47_{(10)} \end{array}$
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b)

$\begin{array}{r} 1101011 \\ + 111010 \\ \hline 10100101 \end{array}$	$\begin{array}{r} 2^6 + 2^5 + 2^3 + 2^1 + 2^0 = 107_{(10)} \\ + 2^5 + 2^4 + 2^3 + 2^1 = 58_{(10)} \\ \hline 2^7 + 2^5 + 2^2 + 2^0 = 165_{(10)} \end{array}$
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c)

$\begin{array}{r} 1011011.11 \\ + 1100111.1 \\ \hline 11000011.01 \end{array}$	$\begin{array}{r} 2^6 + 2^4 + 2^3 + 2^1 + 2^0 + 2^{-1} + 2^{-2} = 91.75_{(10)} \\ + 2^6 + 2^5 + 2^2 + 2^1 + 2^0 + 2^{-1} = 103.5_{(10)} \\ \hline 2^7 + 2^6 + 2^1 + 2^0 + 2^{-2} = 195.25_{(10)} \end{array}$
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d)

$\begin{array}{r} 110111.11 \\ + 1101101.01 \\ \hline 10100101.00 \end{array}$	$\begin{array}{r} 2^5 + 2^4 + 2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2} = 55.75_{(10)} \\ + 2^6 + 2^5 + 2^3 + 2^2 + 2^0 + 2^{-2} = 109.25_{(10)} \\ \hline 2^7 + 2^5 + 2^2 + 2^0 = 165.00_{(10)} \end{array}$
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8. Izračunati izraz $18_{(10)} - 13_{(10)}$ u binarnom brojnem sistemu.

$\begin{array}{r} 18_{(10)} = 10010_{(2)} \\ - 13_{(10)} = 01101_{(2)} \\ \hline \text{jedinični komplement } 10010 \\ + 1 \\ \hline 10011 \text{ dvojni komplement} \end{array}$	$\begin{array}{r} 10010 \\ + 10011 \\ \hline \textcircled{1} 00101 \\ \text{pretek} \Rightarrow \text{rezultat: } 101_{(2)} = 5_{(10)} \end{array}$
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*** Ako je pretek 1 broj je pozitivan; u suprotnom je negativan. Pretek se ne uzima u obzir kad je u pitanju vrijednost broja!!!

9. Izračunati izraz $30.25_{(10)} - 35.5_{(10)}$ u binarnom brojnem sistemu.

$$\begin{array}{r} 30.25_{(10)} = 11110.01_{(2)} \\ - 35.5_{(10)} = 10011.10_{(2)} \\ \hline \end{array}$$

$$\begin{array}{r} \text{jedinični komplement} \quad 011100.01 \\ + \quad 0.01 \\ \hline 011100.10 \quad \text{dvojni komplement} \end{array}$$

$$\begin{array}{r} \\ + \\ \hline \text{negativan} \quad 0111010.11 \\ \text{j.k.} \quad 000101.00 \\ + \quad 0.01 \\ \hline \text{d.k.} \quad -000101.01 \\ \Rightarrow \text{rezultat: } -101.01_{(2)} = -5.25_{(10)} \end{array}$$

10. Pomnoži brojeve $13_{(10)}$ i $5_{(10)}$ u binarnom brojnem sistemu.

$$\begin{array}{r} 13_{(10)} = 1101_{(2)} \\ 5_{(10)} = 101_{(2)} \\ \hline \end{array}$$

$$\begin{array}{r} 1101 \times 101 \\ \hline 1101 \\ 0000 \\ +1101 \\ \hline 100001 = 65_{(10)} \end{array}$$

11. Podijeliti brojeve u binarnom brojnem sistemu:

a) $105_{(10)}$ i $6_{(10)}$

b) $21.75_{(10)}$ i $3_{(10)}$

a)

$$\begin{array}{r} 1101001 : 110 = 10001.1 \\ -110 \\ \hline 01 \\ 0 \\ \hline 010 \\ 0 \\ \hline 0100 \\ 0 \\ \hline 01001 \\ 110 \\ \hline 110 \\ 110 \\ \hline 0 \end{array}$$

b)

$$\begin{array}{r} 10101.11 : 11 = 111.01 \\ -11 \\ \hline 100 \\ 11 \\ \hline 11 \\ 11 \\ \hline 01 \\ 0 \\ \hline 011 \\ 11 \\ \hline 0 \end{array}$$